RADIATIVE–CONDUCTIVE HEAT TRANSFER IN A MEDIUM WITH A CYLINDRICAL GEOMETRY. I

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Exact equations are formulated which describe the radiative–conductive heat transfer in a cylindrical layer of any thickness and approximate solutions to them are then found.

In most studies of compound heat transfer, by both conduction and radiation through a material, one considers a plane layer of semitranslucent material. Only in [1-3] consideration is given to radiative–conductive heat transfer in a medium with a more intricate geometry, namely, a spherical layer in [1] and a cylindrical layer in [2, 3]. In the case of semitranslucent solids the necessity of analyzing the radiative–conductive heat transfer in a cylindrical layer is dictated by the fact that the thermophysical properties at high temperatures are often studied on cylindrical specimens. Compound heat transfer in a nonselective medium has been analyzed in [2] on the basis of the Eddington approximation, considering the mean-over-all-directions radiation intensity and representing the radiation flux in gradient form. Such a model is very approximate and its use is justified only for optically very dense media, as we well know, since in other cases it leads to large errors [4]. In [3] is considered an optically thin layer, for which the original equations can also be greatly simplified. The expression for the radiation vector in [3] does not make it possible, however, to separate the components associated with radiation from the boundary surfaces of the layer and the solution obtained there is applicable when the effect of radiation is small.

The study here will be concerned with steady radiative–conductive heat transfer in a cylindrical layer of any thickness, and the solution will be based on the integral equations of a temperature field. The article will appear in two parts. In Part I we formulate the fundamental equations, which appear much more complicated than in the case of a plane layer so that even a computer-aided solution becomes rather unwieldy. This solution will be shown in Part II, where the temperature fields will also be analyzed. In many cases, however, results can be obtained by a somewhat simpler procedure on the basis of simplified relations. This will be demonstrated here in evaluating the accuracy of the thus obtained values of the spectral properties of a semitranslucent material.

Fundamental Equations. We consider a semitranslucent medium bounded by two coaxial cylindrical surfaces with the respective radii r_1 and r_2 ($r_1 < r_2$) and spectral mirror reflectivities $R_{1\nu}$ and $R_{2\nu}$. The thermophysical and the optical properties of the material are assumed the same at all points in the layer. Solving simultaneously the equation of radiative heat transfer and the equation of energy conservation for the steady state without internal heat sources, we obtain the following nonlinear integral equation of the temperature field (a step-by-step derivation has been given in [5]):

$$\lambda\vartheta(r) = Qr_1 \ln\frac{r}{r_1} - 4\int\limits_{\nu=0}^{\infty}\Big\{[B_2(\nu,\ T_1) - B_2(\nu,\ T_2)][F_1(r) + F_2(r)] + \int\limits_{\rho=r_1}^{\rho=r_2}[j_\nu(r_1) - j_\nu(\rho)]\,K_1(r,\ \rho)\,d\rho\Big\}\,d\nu, \tag{1}$$

with the following designations:

$$F_1(r) = \int\limits_{t=r_1}^{r}\int\limits_{z=0}^{r_1/t}\int\limits_{\psi=0}^{\pi/2}\left\{\exp\left[-\frac{k_\nu(v_2 - v_3)}{\cos\psi}\right] - R_{1\nu}\exp\left[-\frac{k_\nu(v_3 - 2v_1 + v_2)}{\cos\psi}\right]\right\}$$

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$$\times\Bigl\{1-R_{1\nu}R_{2\nu}\exp\Bigl[-\frac{2k_\nu(v_2-v_1)}{\cos\psi}\Bigr]\Bigr\}^{-1}\cos^2\psi d\psi dzdt; \tag{2}$$

$$F_2(r)=\int\limits_{t=r_1}^{r}\int\limits_{z=r_1/t}^{1}\int\limits_{\psi=0}^{\pi/2}\Bigl\{\exp\Bigl[-\frac{k_\nu(v_2-v_3)}{\cos\psi}\Bigr]-\exp\Bigl[-\frac{k_\nu(v_2+v_3)}{\cos\psi}\Bigr]\Bigr\}$$

$$\times\Bigl\{1-R_{2\nu}\exp\Bigl(-\frac{2k_\nu v_2}{\cos\psi}\Bigr]\Bigr\}^{-1}\cos^2\psi d\psi dzdt; \tag{3}$$

$$v_1=\sqrt{r_1^2-t^2z^2};\ v_2=\sqrt{r_2^2-t^2z^2};\ v_3=t\sqrt{1-z^2};\ v_4=\sqrt{\rho^2-t^2z^2};$$

$$K_1(r,\rho)=\int\limits_{t=r_1}^{t=r}K(t,\rho)\,dt; \tag{4}$$

$$K(t,\rho)=\int\limits_{z=0}^{r_1/t}\int\limits_{\psi=0}^{\pi/2}\Bigl\{\Bigl[R_{2\nu}\exp\Bigl(-\frac{k_\nu(2v_2-v_3-v_4)}{\cos\psi}\Bigr)$$

$$\times\Bigl(1-R_{1\nu}\exp\Bigl(-\frac{2k_\nu(v_3-v_1)}{\cos\psi}\Bigr)\Bigr)-R_{1\nu}\exp\Bigl(-\frac{k_\nu(v_3+v_4-2v_1)}{\cos\psi}\Bigr)$$

$$\times\Bigl(1-R_{2\nu}\exp\Bigl(-\frac{2k_\nu(v_2-v_3)}{\cos\psi}\Bigr)\Bigr)\Bigr]\Bigl[1-R_{1\nu}R_{2\nu}\exp\Bigl(-\frac{2k_\nu(v_2-v_1)}{\cos\psi}\Bigr)\Bigr]^{-1}$$

$$-\exp\Bigl(-\frac{k_\nu(v_3-v_4)}{\cos\psi}\Bigr)\Bigr\}\rho\frac{\cos\psi}{v_4}d\psi dz-\int\limits_{z=r_1/t}^{\rho/t}\int\limits_{\psi=0}^{\pi/2}\Bigl\{\exp\Bigl(-\frac{k_\nu(v_3-v_1)}{\cos\psi}\Bigr)$$

$$+\exp\Bigl(-\frac{k_\nu(v_3+v_4)}{\cos\psi}\Bigr)+R_{2\nu}\Bigl[\exp\Bigl(-\frac{k_\nu(v_3+2v_2-v_4)}{\cos\psi}\Bigr)$$

$$+\exp\Bigl(-\frac{k_\nu(v_3+2v_2+v_4)}{\cos\psi}\Bigr)-\exp\Bigl(-\frac{k_\nu(2v_2-v_3+v_4)}{\cos\psi}\Bigr)$$

$$-\exp\Bigl(-\frac{k_\nu(2v_2-v_3+v_4)}{\cos\psi}\Bigr)\Bigr]\Bigl[1-R_{2\nu}\exp\Bigl(-\frac{2k_\nu v_2}{\cos\psi}\Bigr)\Bigr]^{-1}\Bigr\}\rho\frac{\cos\psi}{v_4}d\psi\,dz; \tag{4a}$$

$$(\rho<t);$$

$$K(t,\rho)=\int\limits_{z=0}^{r_1/t}\int\limits_{\psi=0}^{\pi/2}\Bigl\{\Bigl[R_{2\nu}\exp\Bigl(-\frac{k_\nu(2v_2-v_3-v_4)}{\cos\psi}\Bigr)\Bigl(1-R_{1\nu}$$

$$\times\exp\Bigl(-\frac{2k_\nu(v_3-v_1)}{\cos\psi}\Bigr)\Bigr)-R_{1\nu}\exp\Bigl(-\frac{k_\nu(v_3+v_4-2v_1)}{\cos\psi}\Bigr)\Bigl(1-R_{2\nu}$$

$$\times\exp\Bigl(-\frac{2k_\nu(v_2-v_3)}{\cos\psi}\Bigr)\Bigr)\Bigr]\Bigl[1-R_{1\nu}R_{2\nu}\exp\Bigl(-\frac{2k_\nu(v_2-v_1)}{\cos\psi}\Bigr)\Bigr]^{-1}$$

$$+\exp\Bigl(-\frac{k_\nu(v_4-v_3)}{\cos\psi}\Bigr)\Bigr\}\rho\frac{\cos\psi}{v_4}d\psi dz+\int\limits_{z=r_1/t}^{1}\int\limits_{\psi=0}^{\pi/2}\Bigl\{\exp\Bigl(-\frac{k_\nu(v_4-v_3)}{\cos\psi}\Bigr)$$

$$-\exp\Bigl(-\frac{k_\nu(v_3+v_4)}{\cos\psi}\Bigr)-R_{2\nu}\Bigl[\exp\Bigl(-\frac{k_\nu(2v_2+v_3-v_4)}{\cos\psi}\Bigr)$$

$$+\exp\Bigl(-\frac{k_\nu(2v_2+v_3+v_4)}{\cos\psi}\Bigr)-\exp\Bigl(-\frac{k_\nu(2v_2-v_3-v_4)}{\cos\psi}\Bigr)$$

$$-\exp\Bigl(-\frac{k_\nu(2v_2-v_3+v_4)}{\cos\psi}\Bigr)\Bigr]\Bigl[1-R_{2\nu}\exp\Bigl(-\frac{2k_\nu v_2}{\cos\psi}\Bigr)\Bigr]^{-1}\Bigr\}\rho\frac{\cos\psi}{v_4}d\psi dz,$$

$$(\rho>t). \tag{4b}$$

Here the spectral luminance of the boundary surfaces and the radiation coefficients are defined by the equalities $B_2(\nu, T_i) = (1-R_{2\nu})n_\nu^2 I_B(\nu, T_i)$ and $j_\nu(\rho) = k_\nu n_\nu^2 I_B[\nu, T(\rho)]$, respectively. In the derivation of this equation we have assumed both the temperature $T(r_1) = T_1$ and the total energy flux Q incident on the inner boundary surface to be known; the resulting temperature difference $\Delta T = T_1-T_2$ obtained in the course of the solution will then be used for evaluating the effects of various factors.

Evidently the integral equation (1) contains a substantial nonlinearity due to the complicated temperature-dependence of j_ν and B_2. Considering that in practical applications of semitranslucent materials most often $\vartheta \ll T_1$, it becomes possible to linearize Eq. (1). We note that a rigorous evaluation of the linearization error is generally difficult, since no other methods of obtaining reliable estimates are available, except a repeated solution of the nonlinear equation and of the linearized version for various initial parameter values with a subsequent comparison of the results. Such a comparative analysis for a plane layer [6] has shown that, when $\vartheta/T_1 < 0.1$, the linearization error does not exceed a few percent over a wide range of parameter values. Without any further comments, we will now proceed to analyze expressly the linearized equation.

Calculations analogous to those for a plane layer [7] yield

$$\vartheta(r) = \frac{Qr_1}{\lambda}\ln\frac{r}{r_1} - \frac{4}{\lambda}\Delta T\int\limits_{\nu=0}^{\infty}\varepsilon_{2\nu}n_\nu^2\left[\frac{\partial I_B(\nu, T)}{\partial T}\right]_{T_1}[F_1(r)+F_2(r)]\,d\nu$$

$$-\frac{4}{\lambda}\int\limits_{\rho=r_1}^{\rho=r_2}\vartheta(\rho)\left[\int\limits_{\nu=0}^{\infty}k_\nu n_\nu^2\left[\frac{\partial I_B(\nu, T)}{\partial T}\right]_{T_1}K_1(r, \rho)\,d\nu\right]d\rho. \tag{5}$$

For the so-called "gray" approximation, using the spectral properties of the material and of the boundary surfaces averaged over the entire spectrum rather than over the optical range only, we obtain instead of (5)

$$\vartheta(r) = \frac{Qr_1}{\lambda}\ln\frac{r}{r_1} - \frac{16n^2\sigma T_1^3}{\pi\lambda}\varepsilon_2[F_1(r)+F_2(r)]\,\Delta T - \frac{16n^2\sigma T_1^3}{\pi\lambda}k\int\limits_{\rho=r_1}^{\rho=r_2}\vartheta(\rho)K_1(r, \rho)\,d\rho. \tag{6}$$

The applicability of the "gray" approximation to several semitranslucent materials has been demonstrated in [8]. A more thorough comparative analysis of Eqs. (5) and (6) will be made in the last section of this article.

Approximate Relations. The complexity of the resulting integral equations has to do with the unwieldiness of the analytical expressions for their kernels, which is evident from formulas (4a) and (4b). The first approximation to the solution is the free term in Eqs. (5) and (6). In order to give this approximation a physical interpretation, we must note that the last of the terms discarded in (5) and (6) refer to the intrinsic radiation of the medium (they represent the difference between the radiation of the medium at a constant temperature T_1 and of the medium at its true temperature distribution $T(r)$), while the preceding terms refer to the radiation of the boundary surfaces (with absorption and multiple reflections in the layer). Evidently, the more translucent the medium is and the higher its emissivity is, the closer will the free term be to the exact solution.

For an estimate of the first approximation, we again make a comparison with a plane layer.

An analog of Eq. (6) for a plane layer is the equation [7]

$$\alpha(\tau) = a\tau N - N\varepsilon_2\frac{\Delta T}{T_1}F_3(\tau) - N\int\limits_0^{\tau_0}\alpha(\tau')K(\tau, \tau')\,d\tau', \tag{7}$$

where

$$F_3(\tau) = \int\limits_{\varphi=0}^{\pi/2}\cos^2\varphi\sin\varphi\left\{R_1\exp\left(-\frac{\tau_0+\tau}{\cos\varphi}\right) + \exp\left(-\frac{\tau_0-\tau}{\cos\varphi}\right)\right.$$

$$\left. - R_1\exp\left(-\frac{\tau_0}{\cos\varphi}\right) - \exp\left(-\frac{\tau_0}{\cos\varphi}\right)\right\}\left[1 - R_1R_2\exp\left(-\frac{2\tau_0}{\cos\varphi}\right)\right]^{-1}d\varphi; \tag{8}$$

$$\alpha(\tau) = \vartheta(\tau)/T_1;\quad \tau_0 = kH;\quad \tau = kx;\quad N = 8n^2\sigma T_1^3/k\lambda;\quad a = Q/(8n^2\sigma T_1^4);$$

and $K(\tau, \tau')$ is the kernel of the equation – a complicated function of R_1, R_2, and τ_0. Calculations have shown that in many cases discarding the last term in (7) causes no large errors in the determination of the temperature drop ΔT. Thus, for a plate 5 mm thick with $Q = 7500$ W/m^2, $T_1 = 1000$°K, $R = 0.5$, $k = 17.1$ m^{-1}, and $\lambda = 1.5$ W/m·deg the exact value of ΔT is 15.98°C and the first approximation $\Delta T^{(1)} = 16.40$°C. For a plate 18 mm thick ($Q = 6500$ W/m^2, $k = 10$ m^{-1}, $T_1 = 1100$°K, and $R = 0$), the respective

TABLE 1. Temperature Drops in a Plane Layer (Exact solution of Eq. (7) ΔT, first approximation $\Delta T^{(1)}$, $QH/\lambda = 100°C$, $n = 1.46$, $\varepsilon_1 = \varepsilon_2 = \varepsilon$)

T_1, °K	ε	τ_0	ΔT, °C	$\Delta T'$, °C
500	0,062	0,66	99,5	98,3
700	0,076	0,15	98,0	97,0
1000	0,112	0,066	93,6	92,1
1300	0,148	0,043	85,0	83,2
1500	0,192	0,035	75,2	72,9

values are 9.82°C and 10.25°C. A series of calculations was made for a semitranslucent quartz plate 2.4 mm thick covered with platinum foil on both sides. The results shown in Table 1 indicate that for thin layers the error in the value of ΔT based on this approximation does not exceed 2.5% when the emissivity of the boundary surfaces is very low and the temperature is high, so that in this case it is quite unnecessary to use more complicated and unwieldy formulas such as those in [9, 10].

A comparison between the first approximation and the exact solution (which will be analyzed in Part II) leads to the conclusion that the free term in Eqs. (5) and (6) may be used for finding the temperature drop ΔT across a layer, but should not be used for determining the radial temperature profile $T(r)$. A simplified relation for ΔT in the "gray" approximation, for instance, would be

$$\Delta T = \frac{Qr_1}{\lambda}\ln\frac{r_2}{r_1}\left\{1 + \frac{16n^2\sigma T_1^3}{\pi\lambda}\varepsilon_2\left[F_1(r_2) + F_2(r_2)\right]\right\}^{-1}. \tag{9}$$

Temperature drops across a cylindrical layer ($r_1 = 2$ mm, $r_2 = 20$ mm) were calculated according to formula (9) with various values of the reflectivity. The results are shown in Table 2. (It must be remembered that, with the same original parameter values but with radiation disregarded, the value for ΔT was 20°C.) Functions F_1 and F_2, defined by expressions (2) and (3), respectively, were calculated on a model BÉSM-4 computer in not more than 3 min, while a complete solution to the equations on the same computer required 4 h machine time.

On the basis of this latter simplified relation, we will now determine the relative effect of radiation δ_p and δ_c on the heat transfer in a plane and in a cylindrical layer, respectively. Denoting by ΔT_0 the temperature drop across the layer without radiation taken into account, we find

$$\delta_p = \frac{\Delta T_0 - \Delta T}{\Delta T} = \frac{8n^2\sigma T_1^3}{k\lambda}\varepsilon_2 F_3(\tau_0), \tag{10}$$

$$\delta_c = \frac{\Delta T_0 - \Delta T}{\Delta T} = \frac{16n^2\sigma T_1^3}{\pi\lambda}\varepsilon_2\left[F_1(r_2) + F_2(r_2)\right]. \tag{11}$$

Calculations according to (10) and (11) were compared on the basis of the same optical thickness, i.e., with $\tau_0 = kH = k(r_2 - r_1)$. Comparing, for instance, a cylindrical layer ($r_1 = 2$ mm, $r_2 = 20$ mm) with a plate 18 mm thick in the extreme case ($\varepsilon_1 = \varepsilon_2 = 1$ at $T_1 = 1100$°K, $k = 10\ \mathrm{m}^{-1}$, $n = 1.5$, and $\lambda = 1.5$ W/m·deg), we obtained $\delta_p = 6.62$ and $\delta_c = 2.90$. Such a difference is explainable by the fact that, in our formulation of the problem, the most intensely radiating surface (the "hot" wall) of a cylindrical layer has a smaller area than any other cylindrical surface within the layer.

Optical Selectivity Characteristics and the "Gray" Approximation. For a semitranslucent medium with a cylindrical geometry, where the determination of the temperature field involves very unwieldy equations of radiative–conductive heat transfer, any possible simplifying approximation of the spectral optical properties and, particularly, a change from Eq. (5) to Eq. (6) is quite worthwhile. Inasmuch as we are interested in qualitative results rather than in the temperature field specifically, it is permissible in this analysis to consider only ΔT and to determine this quantity by the method shown here.

TABLE 2. Temperature Drop $\Delta T = T_1 - T_2$ in a Cylindrical Layer with Various Reflectivities of the Boundary Surfaces ($T_1 = 1100$°K, $Q = 6500\ \mathrm{W/m^2}$, $\lambda = 1.5$ W/m·deg, $n = 1.46$)

R_1	0	0,5	0,2	0,2
R_2	0	0,5	0,6	0,8
ΔT, °C	5,18	8,33	8,57	11,3

TABLE 3. Piecewise-Constant Approximation of the Absorption Spectrum of KV Quartz Glass at Temperatures of 1100°K

Boundary interval of wavelengths, μ	0,25	2,5	2,674	2,874	3,289	4,587
	2,5	2,674	2,874	3,289	4,587	5,0
K, m^{-1}	5	29	120	44	255	1410

For a feasibility study of such a changeover to the "gray" approximation, it is worthwhile to use a material whose absorption characteristics vary appreciably within the region of the spectrum adjoining the peak of thermal radiation. Such a material is, for example, fused quartz at a temperature of 1100°K [11]. The absorption in SiO_2 within the 0.25-2.5 μ wavelength band is so weak that it can be measured only very inaccurately. Within the 2.5-5.0 μ wavelength band k_ν increases by 2-2.5 orders of magnitude, and beyond those wavelengths it increases very rapidly almost to infinity. Taking into account such a complex trend of the $k(\nu)$ curve, the latter was replaced by a piecewise-constant approximation shown in Table 3 (all measurements of the spectral absorption coefficient were made by Settarova [11]). On the basis of these data we calculated ΔT. In the course of these calculations we have found that for the spectrum "window," i.e., for the 0.25-2.5 μ band, it is advisable to use the limit values of functions F_1 and F_2:

$$\lim_{k\to 0}[F_1(r)+F_2(r)]=\frac{\pi}{4}\cdot\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\,r_1\ln\frac{r}{r_1}.$$

For the case $\varepsilon_1 = \varepsilon_2 = 1$ (r_1 = 2 mm, r_2 = 20 mm, Q = 6500 W/m^2, λ = 1.5 W/m·deg, and n = 1.46), Eq. (5) yielded ΔT_C = 9.76°C.

Before making a comparison with the "gray" approximation, it is necessary to find the mean spectral absorption coefficient. As we well known [4], one uses either the Planck mean coefficient (k_P) or the Rosseland mean coefficient (k_R). Calculations have shown that for a material like fused quartz both coefficients are rather indeterminate. This has to do with the fact that the integration over all wavelengths from 0 to ∞, according to the definition of k_P and k_R, is in practice replaced by integration over a finite range of the spectrum, the choice of which is quite arbitrary, with the high range of k_ν values strongly affecting k_P and the low range of k_ν values strongly affecting k_R. If k_P is calculated on the basis of the 2.5-4.587 μ band, within which k_ν changes from 5 to 1410 m^{-1}, then we obtain k_P = 125.5 m^{-1}. Adding the 0.25-2.5 μ band ($k_\nu \approx 5\ m^{-1}$) will change the calculated value of k_P by less than 1%. Adding the 4.587-10.0 μ band with the underestimated value k_ν = 1410 m^{-1}, on the other hand, will increase the calculated value of k_P to 710 m^{-1}. In the case of k_R the tendency is just opposite: the 2.5-4.587 μ band yields k_R = 163.4 m^{-1}, the 2.5-10.0 μ band yields k_R = 161.2 m^{-1}, and the 0.25-4.587 μ band yields k_R = 10.1 m^{-1}. (In these calculations the $k(\nu)$ curve has been approximated more accurately by straight and parabolic segments.) Having determined the temperature drop from (9) with k_P = 125.5 m^{-1}, we find ΔT_P = 13.4°C. The same formula with k_R = 10.1 m^{-1} yields ΔT_R = 7.02°C. Evidently, neither of these two "gray" approximations agree with ΔT_C = 9.76°C according to formula (5).

Inasmuch as the inadequacy of the "gray" approximation is due primarily to the existence of the "window" in the spectrum, we will instead transform Eq. (6) in the following manner:

$$\vartheta(r)=\frac{r_1}{\lambda}\ln\frac{r}{r_1}\left\{Q-\pi\varepsilon_n n^2\Delta T\int_{\nu_1}^{\nu_2}\left[\frac{\partial I_B(\nu,T)}{\partial T}\right]_{T_1}d\nu\right\}$$

$$-\frac{16n^2\sigma T_1^3}{\pi\lambda}\varepsilon_2\Delta T[F_1(r)+F_2(r)]-\frac{16n^2\sigma T_1^3k}{\pi\lambda}\int_{r_1}^{r_2}\vartheta(\rho)K_1(r,\rho)\,d\rho, \qquad (6a)$$

assuming k_ν = 0 within the translucency band (ν_1, ν_2) of the material and calculating the mean absorption coefficient k on the basis of the rest of the spectrum. Letting k = k_P = 125.5 m^{-1} yields $\Delta T'_P$ = 8.9°C, but letting k = k_R = 163.4 m^{-1} yields $\Delta T'_R$ = 9.62°C, the latter being much closer to ΔT_C than before.

Thus, even in an unfavorable situation it is possible by this artifice to improve the accuracy of the "gray" approximation and to simplify the procedure for determining the temperature field. More accurate results are obtained, furthermore, if the weak-absorption band is omitted and the mean coefficient is based on the rest of the spectrum according to Rosseland.

NOTATION

λ	is the thermal conductivity;
k	is the absorption coefficient;
n	is the refractive index;
k_P	is the Planck mean absorption coefficient;
k_R	is the Rosseland mean absorption coefficient;
j	is the radiation coefficient of the medium;
R	is the reflectivity;
ε	is the emissivity;
$\varepsilon_n = \varepsilon_1\varepsilon_2(\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2)^{-1}$	is the referred emissivity;
ν	is the spectral frequency;
$B(\nu, T)$	is the spectral luminance of the boundary surfaces;
Q	is the total energy flux;
$I_B(\nu, T)$	is the spectral radiation intensity of an ideal black body;
σ	is the Stefan constant;
$\vartheta(r) = T_1 - T(r)$	is the relative temperature;
$\Delta T = T_1 - T_2$	is the temperature drop across a layer;
τ_0	is the optical thickness;
H	is the thickness of a plane layer;
r and ρ	are the cylindrical coordinates.

Subscripts

1 and 2	denote the inner and the outer surface of a layer, respectively.
ν	denotes the spectral parameter.

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